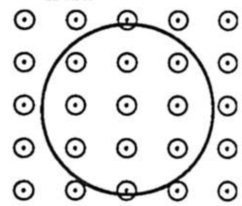
# MULTIPLE CHOICE SOLUTIONS--E&M

## TEST II




**1.)** A 50 turn coil whose face area is .5 square meters and whose resistance is $R = 10\ \Omega$ faces a uniform **B**-field coming out of the page that changes at a rate of -.4 teslas per second. At $t = 0$, the magnetic field intensity is .25 teslas. The magnitude of the current induced in the coil will be:

**a.) 1 amp.** [The induced EMF $\varepsilon$ in the coil is related to the current $i$ and the resistance $R$ by $\varepsilon = iR$. We know the resistance. We need the induced EMF. To get that, we use Faraday's Law. Noting that the angle between the **B**-field and the coil's area vector is zero (the two are both out of the page), we can write:

$$\varepsilon = -N\frac{\Delta\phi_m}{\Delta t}$$

$$\Rightarrow \quad \varepsilon = -N\frac{\Delta(BA\cos 0^\circ)}{\Delta t}$$

$$\Rightarrow \quad \varepsilon = -NA\frac{\Delta B}{\Delta t}$$

$$\Rightarrow \quad \varepsilon = -(50)(.5\ m^2)(-.4\ teslas/second)$$

$$\Rightarrow \quad \varepsilon = 10 volts.$$

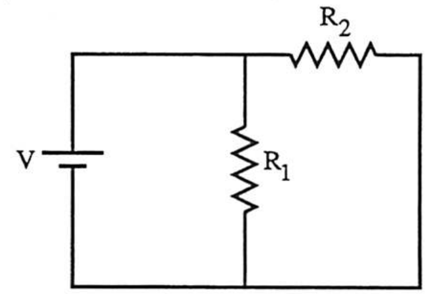
Dividing the EMF by $R$ yields *(10 volts)/(10 Ω) = 1 amp*. This response is true.]

b.) 2 amps. [Nope.]

c.) 4 amps. [Nope.]

d.) None of the above. [Nope.]

**2.)** If $R_1$ is doubled:



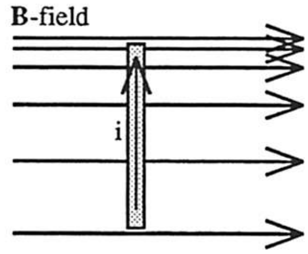

a.) The power dissipated by $R_2$ will decrease. [Power is defined as *work per unit time*. In the case of an electrical device, $P = i^2R$ (this is usually used when dealing with the power dissipated by a resistor), $P = iV$ (this is usually used when dealing with the power provided to a system by a battery or power supply), and $P = V^2/R$ (this is rarely used except when multiple choice test designers are trying to confuse students with rarely used formulas). When dealing with a resistor, the *current* determines how the power function will act. Why? The current is *squared* in the power relationship. If you halve the resistance of a resistor and its current doubles, $i^2R$ suggests that the power dissipated by the resistor will *double* (try the math if you don't believe me). In this case, doubling $R_1$ will do nothing to $R_2$ ($R_2$'s voltage hasn't changed so its current remains the same, and the power dissipated by $R_2$ will not change. This response is false.]

**b.) The power dissipated by $R_1$ will halve.** [Assume the initial power was $i^2R_1$. When $R_1$ doubles, the current through $R_1$ halves. The power dissipated by $R_1$ becomes $(i/2)^2(2R_1) = (1/2)i^2R_1$. Evidently, the power halves and this response is true.]

c.) The power dissipated by $R_1$ will quarter. [Nope.]

d.) The power dissipated by $R_1$ will stay the same. [Nope.]

e.) None of the above. [Nope.]



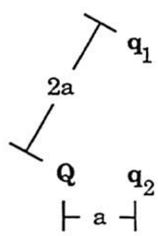

**3.)** A current-carrying wire is placed in a varying magnetic field as shown. Assuming gravity is zero and the wire is free to move as it will:

a.) The wire will accelerate to the right and spin clockwise relative to the plane of the paper. [Using $\boldsymbol{F} = i\boldsymbol{L}x\boldsymbol{B}$ at the lower end of the wire, the direction of the magnetic force will be into the page. As the current in the upper part of the wire is in the same direction, it will also produce a force into the page. With the net force into the page, the wire will accelerate in that direction and this response is false.]

**b.) The wire will accelerate into the page and spin so that its top rotates *into* the page.** [This response has the correct direction for the force, but does it have the correct response for rotation? As for that, the magnitude of the force at the upper end of the wire will be greater than the magnitude of the force at the lower end (the field is greater at the top than at the bottom). That means that relative to the wire's center of mass, the torque produced by the force at the upper end will be greater than the torque produced by the force at the lower end. The net effect will be a rotation of the wire with its top directed *into* the page. As such, this response is true.]

c.) The wire will accelerate out of the page and spin so that its top rotates out of the page. [Nope.]

d.) None of the above. [Nope.]



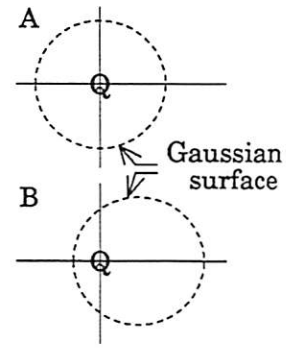

**4.)** Two equal charges $q_1$ and $q_2$ (they have been labeled differently for identification purposes) are placed as shown in the vicinity of a third charge $Q$. We know that the force magnitude $q_1$ applies to $Q$ is $F_1$, and the force magnitude $q_2$ applies to $Q$ is $F_2$. The ratio of $F_1$ to $F_2$ is:

a.) 1 to 2. [Coulomb's Law states that the magnitude of the force a point charge feels due to the presence of a second point charge is equal to $\frac{1}{4\pi\varepsilon_o}\frac{Q_nQ_m}{r^2}$. From this, if the distance between two charges is doubled, the force diminishes by a factor of 4. Because $q_1$'s distance to $Q$ is twice that of $q_2$'s, $F_1$ will be one-quarter of $F_2$ and the ratio of $F_1/F_2 = (1/4)/1 = 1/4$. This response is false.]

b.) 2 to 1. [This won't do.]

**c.) 1 to 4.** [This is the one.]

d.) 4 to 1. [This is tricky. If you write out the force equation $\frac{1}{4\pi\varepsilon_o}\frac{Q_nQ_m}{r^2}$ for each charge combination, you will find that $F_1/F_2 = 1/4$, not $F_1/F_2 = 4/1$. This response is false.]




**5.)** A point charge $Q$ is placed at the origin of a coordinate system, and a Gaussian surface is placed around the charge as shown in *sketch A*. A second, identical situation is set up with the exception that the Gaussian surface is positioned as shown in sketch B.

a.) The electric flux through the two surfaces will be different. [According to Gauss's Law, the total electric flux is related solely to the amount of charge enclosed inside the Gaussian surface. As the charge enclosed is the same in both cases, the net electric flux will be the same in both cases. This response is false.]

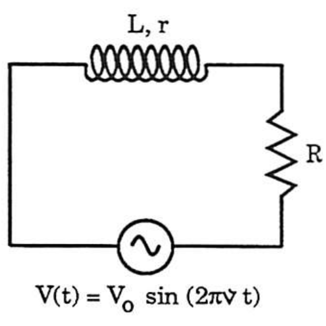
b.) The electric flux through the surface in *sketch A* will equal $Q/\varepsilon_o$, and the magnitude of the electric field along the *x axis* in *sketch A* will be $E = \frac{Q}{4\pi\varepsilon_o x^2}$. [According to Gauss's Law, the electric flux (i.e., the left side of Gauss's equation) will, indeed, equal $Q/\varepsilon_o$. As the electric field will be that of a point charge, the electric field down the axis will also be $E = \frac{Q}{4\pi\varepsilon_o x^2}$. This response is true. Are there other true responses?]

c.) The electric flux through the surface in *sketch A* will be $Q/\varepsilon_o$, and the magnitude of the electric field along the *x axis* in *sketch B* will be $E = \frac{Q}{4\pi\varepsilon_o x^2}$. [Just because the Gaussian surface is off center doesn't mean that the net electric flux through its surface isn't going to equal $Q/\varepsilon_o$, as Gauss's Law maintains. And even though the field intensity value will be different at different points on the off-set Gaussian surface, the electric field down the *x axis* is still going to be that of a point charge, or $E = \frac{Q}{4\pi\varepsilon_o x^2}$. This response is true.]

d.) All of the above. [Nope.]

**e.) Only *b* and *c* are true.** [This is the one.]

f.) None of the above. [Nope.]



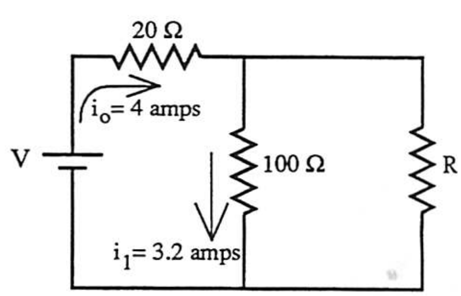

**6.)** The inductance of the inductor in the circuit shown is 10 mH and its resistor-like resistance is 15 Ω. The load resistor is $R = 1000\ \Omega$. At 200 cycles/second, the inductive reactance of the circuit is:

**a.) 12.6 Ω.** [The circuit's inductive reactance is $X_L = 2\pi\nu L = (2\pi)(200\ Hz)(10x10^{-3}\ H) = 12.56\ \Omega$. This response is true.]

b.) .08 Ω. [If you inverted the answer, you got this incorrect response.]

c.) .008 Ω. [Nope.]

d.) None of the above. [Nope.]




**7.)** Consider the circuit shown.

a.) The unknown resistor $R = 25\ \Omega$, and the voltage $V = 80\ volts$. [DON'T BE LED BY THE NOSE. Just because $R$ is the first thing asked about doesn't mean that determining it will be the easiest part of the problem. In fact, finding the voltage $V$ is definitely easier. What's more, this problem is more easily done by the seat of your pants than by using

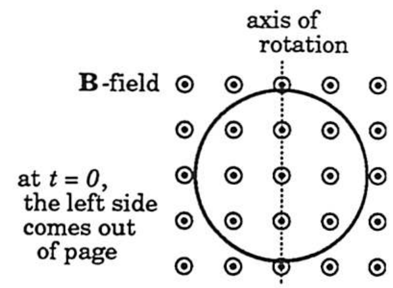
Kirchoff's Laws. Starting with the voltage, all you need to notice is that the voltage drops across the 20 Ω and 100 Ω resistors have to add to the voltage across the battery. Using that information, we can write $V = (20\ \Omega)(4\ amps) + (100\ \Omega)(3.2\ amps) = 400\ volts$. From that information alone, this response is false.]

b.) The unknown resistor $R = 400\ \Omega$, and the voltage $V = 200\ volts$. [Once again, the voltage is wrong and this response is false. Notice that by observing this, you have eliminated two of the possible four responses. Even if you didn't have time to do the problem completely, this would considerably increase your chances should you try to guess.]

**c.) The unknown resistor $R = 400\ \Omega$, and the voltage $V = 400\ volts$.** [We have the right voltage, so now we have to determine $R$. Let's assume you were hoodwinked into dealing with the unknown resistance first (i.e., assume you don't know $V$ for this part). How would you determine $R$? The current through $R$ is $(4\ amps) - (3.2\ amps) = .8\ amps$. The voltage across the 100 Ω resistor and $R$ have to be the same, so you could write $(100\ \Omega)(3.2\ amps) = (.8\ amps)R$. Solving yields $R = 400\ \Omega$. This response is true.]

d.) None of the above. [Nope.]

**8.)** A spinning coil whose axis is in the $+j$ direction sits in a **B**-field that comes out of the page. The coil's rotation is such that at $t = 0$, it is in the plane of the page with its left side moving *out of* the page.



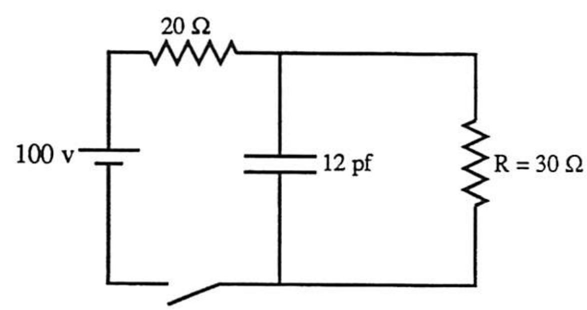

a.) The flux change is a maximum at $t = 0$, and the induced current's direction is clockwise. [If you spin a coil, you will find that there is very little effective area change, relative to the magnetic field, as the coil rotates through the position alluded to in this response. In fact, $\frac{d\phi_m}{dt}$ at $t = 0$ is *zero*. As this is the case, this response is false.]

b.) The flux change is a maximum at $t = 0$, and the induced current's direction is counterclockwise. [From above, this response is false.]

c.) The flux change is zero at $t = 0$, and the induced current's direction just an instant after $t = 0$ is clockwise. [As stated above, the flux change at the instant mentioned is zero, so the first part of this response is true. As for the second part, the fact that the flux is decreasing with time just after $t = 0$ means that the induced magnetic field will be *with* the external field *out of the page*. The induced current that will set up such a magnetic field through the coil's face is in the counterclockwise direction. This response is false.]

**d.) The flux change is zero at $t = 0$, and the induced current's direction just after $t = 0$ is counterclockwise.** [This is the one.]

**9.)** At $t = 0$, the switch is closed. If the capacitors are initially uncharged, what is the initial current through $R$?

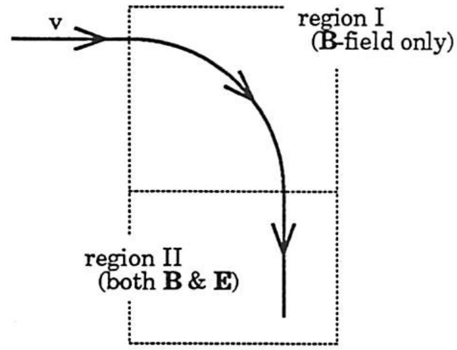



a.) 5 amps. [This is a particularly sneaky problem, especially given the fact that there is no hint as to the correct answer in the selections offered. When the switch is closed, there will be *no voltage* across the capacitor (it is uncharged). As the capacitor is in parallel with the 30 Ω resistor, there will be no initial voltage across that resistor, either. As such, the initial current through that resistor will be zero. Looking at it in a little more qualitative way: If the capacitor initially acts like a short, through which path would you expect current to take, through the short or through a resistor? The short will take it every time, and this response is false.]

b.) 2 amps. [Nope.]

c.) 1 amp. [Nope.]

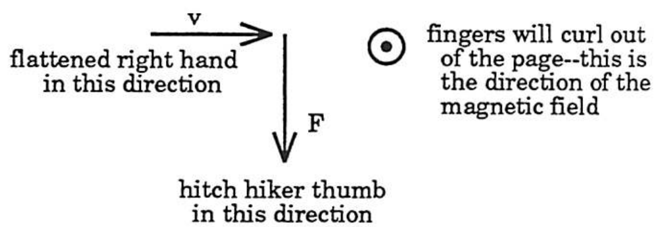
**d.) None of the above.** [This is the one.]




**10.)** A negative charge moves with known velocity magnitude $v$ into *region I* in which exists an unknown **B**-field. It accelerates as shown in the sketch, then enters *region II* in which there exists not only $\boldsymbol{B}$ but also an unknown electric field $\boldsymbol{E}$.

a.) The direction of the **B**-field is toward the bottom of the page, and as there is no need for the presence of an electric field to keep the charge moving in the direction shown in *region II*, $E = 0$. [There clearly is an initial downward force on the charge as it enters *region I*, but magnetic *forces* do not orient themselves in the direction of magnetic *fields*. The direction of a magnetic force will be PERPENDICULAR to both the direction of $\boldsymbol{B}$ and the direction of $\boldsymbol{v}$. In short, if $\boldsymbol{B}$ had been directed toward the bottom of the page, the force $q\boldsymbol{v}x\boldsymbol{B}$ would have pushed the charge *into the page*. This response is false.]

**b.) The direction of the B-field is into the page, and the direction of the E-field is to the left.** [Use the cross product associated with $q\boldsymbol{v}x\boldsymbol{B}$ backwards to determine the direction of $\boldsymbol{B}$. Do this on the assumption that $\boldsymbol{q}$ is positive (it is easiest to do the problem for a positive charge, then simply reverse the direction of $\boldsymbol{B}$ to accommodate for the fact that it is really a *negative* charge you are dealing with). That is, run your flattened right hand in the direction of the initial $\boldsymbol{v}$, then orient your thumb hitch hiker style in the direction of the required force (in this case, downward). The direction in which your curled fingers end up is the direction of the magnetic field. The direction that satisfies this criterion for this problem is out of the page, but that assumes a positive charge. For a negative charge, the magnetic field direction must be *into the page* and the first part of this response is true. What about the second part? With a **B**-field directed into the page, a negative charge moving downward in the vertical will feel a magnetic force oriented to the left (doing the cross product using $q\boldsymbol{v}x\boldsymbol{B}$ produces a force on a positive charge to the right--reverse that to get the direction for a negative charge). If a positive charge felt a force to the left, and if you wanted an electric field to counteract that force, the direction of that field would have to be to the right. As the real charge in this problem is negative, the direction of the electric field we need must be opposite that direction, or to the left. This response is true.]



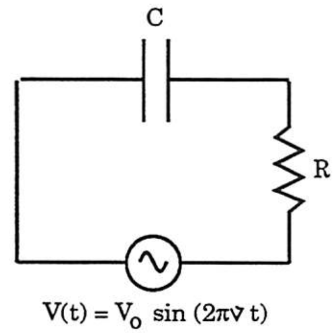

c.) The direction of the **B**-field is into the page, and the direction of the **E**-field is to the right. [Nope.]

d.) The direction of the **B**-field is out of the page, and the direction of the **E**-field is to the left. [Nope.]

e.) The direction of the **B**-field is out of the page, and the direction of the **E**-field is to the right. [Nope.]

f.) None of the above. [Nope.]

11.) The capacitance of the capacitor in the circuit shown is $C = 10\ \mu f$. The load resistor is $R = 100\ \Omega$. The frequency of the source is 80 cycles/second. The approximate impedance of the circuit is:




a.) 100 Ω. [In its most general form, the impedance expression is $Z = \sqrt{R_{net}^2 + (X_L - X_C)^2}$ *ohms*, where $X_L = 2\pi\nu L$ *ohms* (the inductance $L$ has to be in *henrys*, not *mH*, etc.) and $X_C = \frac{1}{2\pi\nu C}$ *ohms* (the capacitance $C$ has to be in *farads*, not *mf* or *pf*, etc.). In this problem there is no inductance. As such, the impedance expression becomes

$$\sqrt{R_{net}^2 + \left(\frac{1}{2\pi\nu C}\right)^2} = \sqrt{(100\ \Omega)^2 + \left(\frac{1}{2\pi(80\ hz)(10x10^{-6}\ henrys)}\right)} = 223\ \Omega.$$ Response is false.]

**b.) 220 Ω.** [This is the one.]

c.) 300 Ω. [Nope.]

d.) None of the above. [Nope.]

12.) A .25 kg mass has a 2 coulomb charge on it. It is placed at *Point A* in an electric field and released from rest, freely accelerating over a .4 meter distance to *Point B* where its velocity is observed to be 8 m/s. The electrical potential of *B* is 40 volts.

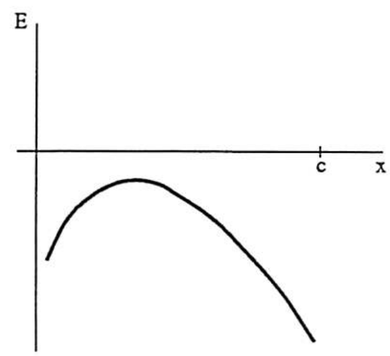
**a.) The electrical potential at *A* is 44 volts.** [The easiest way to do this problem is to use the conservation of energy. The initial kinetic energy is zero (it starts from rest), the initial potential energy is $qV_A$, no extra work is done between *A* and *B*, the final kinetic energy is $.5mv^2$, and the final potential energy is $qV_B$. Putting it all together, we get: $qV_A = .5mv^2 + qV_B$, or $V_A = [1/q][.5mv^2 + qV_B] = [1/(2\ coul)][.5(.25\ kg)(8\ m/s)^2 + (2\ coul)(40\ volts)] = 44\ volts$. This response is true.]

b.) The electrical potential at *A* is 36 volts. [If you ignored the fact that the mass has kinetic energy at *B*, you got this incorrect response.]

c.) The electrical potential at *A* is 40.5 volts. [If you didn't square the velocity term in the conservation of energy expression, you got this incorrect response.]

d.) None of the above. [Nope.]

13.) *Charge A* is placed at the origin and *charge B* is placed a distance *c* units down the *+x axis*. Data is taken between the charges and a plot of the subsequent electric field *E* as a function of position *x* is graphed.




a.) Both charges are positive, and *charge A* is larger in magnitude. [If both charges are positive, there will be a point between the two where the electric field is zero. As that isn't the case, this response is false.]

b.) *Charge A* is positive while *charge B* is negative, and *charge A* is larger in magnitude. [If *charge A* is positive, the electric field close to the origin will be in the positive direction. As this graph is solely in the negative region, this response is false.]

**c.) *Charge A* is negative while *charge B* is positive, and *charge B* is larger in magnitude.** [With *charge A* negative, the electric field close to the origin will be in the negative direction, which is the case with our graph. With *charge B* positive, the field close to it will be in the $-x$ direction (repulsion on an object that is to the left of a field producing positive charge will push a *positive test charge* to the left, or in the $-x$ direction). In short, we have the right combination as far as the charge sign goes. As to the question of which is larger: As we move from the origin in the $+x$ *direction*, the electric field due to *charge A* will decrease as $1/r^2$ until *charge B's* field contribution offsets the decrease. When that happens, the field *magnitude* will begin to get larger in a negative sense (i.e., the field values will move farther and farther away from zero). If *charge A* is the larger of the two charges, its field will range considerably beyond the halfway point before *charge B's* contribution will make much difference. In fact, our graph shows a difference before the halfway point, so *charge B* must be larger than *charge A*. This response is true.]

d.) *Charge A* is negative while *charge B* is positive, and *charge B* is smaller in magnitude. [Nope.]

e.) None of the above. [Nope.]

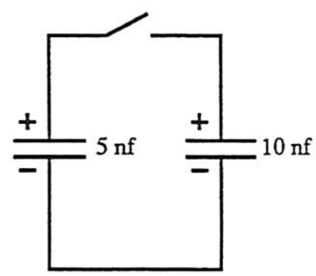
**14.)** A solid sphere has a volume charge density of $kr^4$, where $k$ is a constant with appropriate units. If the sphere's radius is $a$:

a.) The electric field is smaller at $a/2$ than it is at $a$, and it is smaller at $a$ than it is at $2a$. [This particular volume charge density function increases as $r^4$. Furthermore, the volume of an expanding Gaussian sphere increases as $r^3$. So even though the electric field of a point charge drops off as $1/r^2$, the farther we get from the center of this system, the proportionally greater the charge enclosed becomes. As such, the field at $a$ should be larger than at $a/2$ (i.e., the first part of this selection is correct). As for outside the sphere, a Gaussian surface will include all of the charge inside the sphere for radius $a$ and radius $2a$. With the same charge enclosed, the electric flux through each surface will be the same, but because the surface area of the Gaussian surface evaluated at $r = a$ is smaller than its counterpart at $r = 2a$, the field at $r = a$ must be larger. This response is false.]

**b.) The electric field is smaller at $a/2$ than it is at $a$, and it is greater at $a$ than it is at $2a$.** [From above, this response is true.]

c.) The electric field is greater at $a/2$ than it is at $a$, and it is smaller at $a$ than it is at $2a$. [This response is false.]

d.) The electric field is greater at $a/2$ than it is at $a$, and it is greater at $a$ than it is at $2a$. [This response is false.]



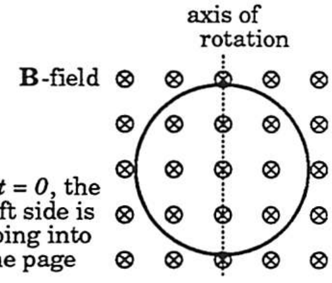

**15.)** A 5 nf capacitor (call it $C$) is charged by a 50 volt power supply and then isolated. A 10 nf capacitor (call it $2C$) is charged by a 100 volt power supply and then isolated. The two capacitors are then connected as shown. At $t = 0$, the switch is closed. After a long period of time, the voltage across the 5 nf capacitor will be:

**a.) 83.4 volts.** [When the switch is closed, the charge in the system will redistribute itself until the voltage across each capacitor is the same (that means that if we can determine the voltage across *either* capacitor, we will have answered our question). Knowing this voltage relationship, we can write $V = Q_{1,new}/C = Q_{2,new}/(2C)$, or $2Q_{1,new} = Q_{2,new}$. The total charge in the system will be the sum of the charge on each positive plate *before* the plates are connected (once connected, where is the charge going to go but inside the system?). That means $Q_{tot,old} = Q_{1,old} + Q_{2,old} =$

$CV_1 + (2C)V_2 = (5x10^{-9}\ f)(50\ v) + (10x10^{-9}\ f)(100\ v) = 1.25x10^{-6}$ *coulombs*. That total charge will redistribute itself in a ratio of 1:2 ($2Q_{1,new} = Q_{2,new}$ means there is twice the charge on $Q_{2,new}$ than on $Q_{1,new}$, hence the 1:2 ratio . . . versus 2:1). Using that ratio, we get $Q_{1,new} = .417x10^{-6}$ *coulombs* and $Q_{2,new} = .833x10^{-6}$ *coulombs*. With that information, the voltage across the 5 nf capacitor will be $V = Q_{1,new}/C = (.417x10^{-6}\ coulombs)/(5x10^{-9}\ f) = 83.4$ *volts*. This response is true.]

b.) 75 volts. [Nope.]
c.) 66.6 volts. [Nope.]
d.) None of the above. [Nope.]

**16.)** A coil is rotated in a fixed **B**-field that is oriented into the page. The angular position of the coil at $t = 0$ is shown to the right with the coil's left side moving into the page. In the position shown:



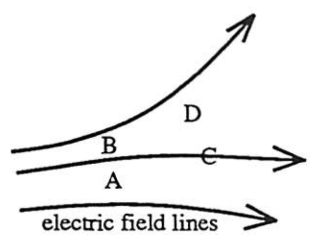

a.) The induced current in the coil will be in the counterclockwise direction, relative to the sketch, and the current will be increasing. [The change of flux at $t = 0$ is zero, so there will be no induced current in the circuit at that point in time. This response is false.]

b.) The induced current in the coil just after $t = 0$ will be in the clockwise direction, relative to the sketch, and the current will be decreasing. [Just after $t = 0$, the flux will be decreasing. The induced current will set itself up so as to produce an induced magnetic field through the coil that is *in the direction of the external field*. The current direction that will effect that situation will be clockwise. This part of this response is true. Moving on, since the current was zero and is now non-zero, the current must be increasing and this response is false.]

c.) The induced current in the coil just after $t = 0$ will be in the counterclockwise direction, relative to the sketch, and the current will be decreasing. [Wrong induced current direction. This response is false.]

**d.) The induced current in the coil just after $t = 0$ will be in the clockwise direction, relative to the sketch, and the current will be increasing.** [From above, this response is true.]

**17.)** Equal positive charges are placed at *Points A, B, C,* and *D* in the **E**-field shown. When released, which charge will probably travel the farthest in 10 seconds?




**a.) The charge at *Point A*.** [This is a little tricky. The positive charge will accelerate in the general direction of the field lines in its vicinity (the direction of each electric field line is defined as the direction a *positive charges* will accelerate if placed in the field on the line). The fact that *Point C* is directly on a line means nothing (it is the distance *between* lines that denotes field strength). The positive charge put at *Point B* will start out in the largest electric field (the lines are closest there), but as it accelerates to the right and upward, it will quickly find itself in an ever diminishing field. The charge placed at *Point A* will be in a field that is not quite as strong as at *Point B*, but as the charge accelerates to the right the field remains fairly constant and moderately large. The charge at *Point C* will be in almost the same situation as the charge at *A*, but it will start out in a little smaller field and, over time, will have experienced a lesser acceleration. As the time interval is relatively large, and given the diminishing field the charge at *Point B* will experience, the charge at *Point A* will travel the farthest. In short, this response is true.]

b.) The charge at *Point B*. [Nope.]

c.) The charge at *Point C*. [Nope.]

d.) The charge at *Point D*. [Nope.]

**18.)** A wire's radius is doubled. With that change, the power dissipated by a given length will:

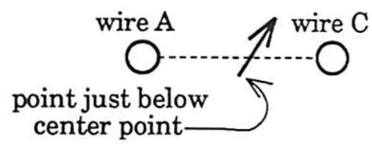
a.) Quarter. [Current is essentially a measure of charge flow (i.e., the amount of charge that passes by a particular point per unit time). When the radius of a wire is doubled, its cross sectional area goes up by a factor of $(2)^2 = 4$ (the area of a circle is $\pi r^2$). That being the case, one would expect that the current would go up by a factor of four, and this response is false.]

b.) Halve. [Nope.]

c.) Double. [Nope.]

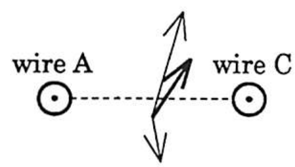
**d.) Quadruple.** [This is the one.]

e.) None of the above. [Nope.]




**19.)** Two current-carrying wires are oriented perpendicular to the page. Their current magnitudes and directions are unknown. The net magnetic field just below the center point of a line passing between the charges is shown in the sketch.

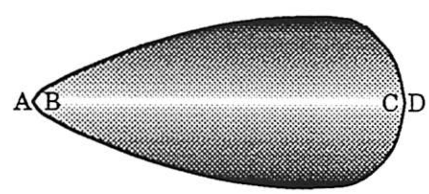
a.) *Wire A's* current must be into the page with *wire C's* current being out of the page, and *A's* current magnitude must be larger than *C's*. [This is another example of a problem in which you simply have to diddle around with various combinations of the *right thumb rule* until you get a current combination that yields a net magnetic field in the desired direction. The auxiliary sketch shows the correct configuration. Note that the current in *A* must be larger than the current in *C*. This response is false.]




**b.) Both *wires A* and *C* must have currents out of the page, and *A's* current magnitude must be larger than *C's*.** [This is the one.]

c.) *Wire A's* current must be out of the page with *wire C's* current being into the page, and *A's* current magnitude must be smaller than *C's*. [Nope.]

d.) Both *wire A's* current and *C's* current must be into the page, and *A's* current magnitude must be smaller than *C's*. [Nope.]



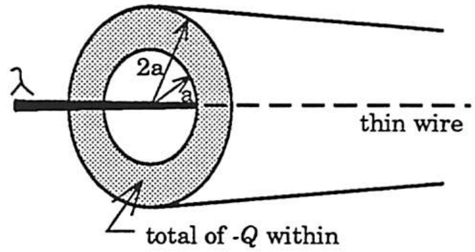

**20.)** The thin-shelled, egg-shaped conductor has a total charge *Q* placed on it. If points *B* and *C* are *inside* the shell's surface:

a.) The electric potential at *B* is greater than at *C*, and the electric field at *A* is greater than at *D*. [Because the electric field inside a conductor is zero, the electrical potential inside and on the surface of a conductor is a constant. That means that even though the egg's curvature is different close to *Point B* and *Point C*, the voltage will be the same nevertheless. On the first count, therefore, this statement is false.]

**b.) The electric potential at *B* is the same as at *C*, and the electric field at *A* is greater than at *D*.** [The first statement is true, given what has been said above. As for the second part: Due to shielding, the electric field will be greatest where the curvature is greatest. As such, the electric field at *Point A* will be greater than the electric field at *Point D*, and this statement is true. Note that this isn't inconsistent with the voltage being the same over the shell's surface. The electric field is related to the *rate of change* of the voltage. The voltage value will be the same at every point on the conductor's surface, but its *rate of change* will differ at different points as one proceeds out from the conductor.]

c.) The electric potential at *B* is greater than at *C*, and the electric field at *A* is the same as at *D*. [Nope.]

d.) The electric potential at *B* is smaller than at *C*, and the electric field at *A* is smaller than at *D*. [Nope.]



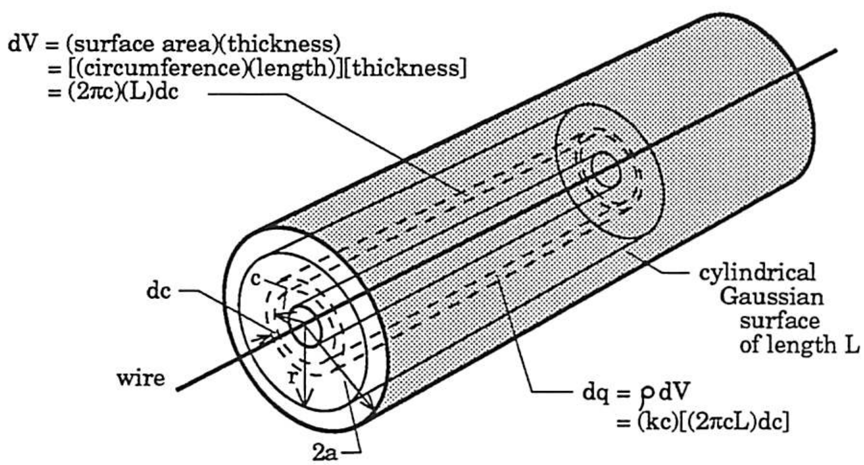

**21.)** A long, hollow cylinder of inside radius $a$ and outside radius $2a$ has a thin wire with a linear charge density $\lambda$ running down its central axis. Additionally, for every meter of length, a negative charge $-Q$ is uniformly sprinkled throughout the volume between $a$ and $2a$. It is observed that at $(5/4)a$, the electric field is zero. The linear charge density $\lambda$ must equal:

**a.) Approximately .188Q coulombs per unit length.**

[There is an easy way and a hard way to do this problem. First, the easy way. We know that for the electric field to be zero at $5a/4$, $q_{enclosed}$ inside the Gaussian surface of radius $5a/4$ must be zero. Let's assume we have a one meter Gaussian cylinder whose radius is $5a/4$. The charge enclosed due to the *wire* will be $\lambda(1\ meter) = \lambda$. The charge enclosed due to the cylinder will equal the product of the charge per unit length $-Q$ along the cylinder times one meter times the fraction of cylinder's volume between $a$ and $5a/4$. As that volume is

$$\frac{\pi(5a/4)^2(1\ meter) - \pi a^2(1\ meter)}{\pi(2a)^2(1\ meter) - \pi a^2(1\ meter)}$$

that charge becomes (-3Q/16). For $q_{encl}$ to equal zero, the magnitude of that *charge enclosed* value must equal $\lambda$. The alternate way of doing this is to use Gauss's Law. To start, we need the volume charge density function for the cylinder. The normal way to do this is to divide the charge by the volume enclosing the charge. We know that one meter of cylinder has $-Q$ worth of charge in it (i.e., the charge/length is $-Q$ *coulombs/meter*). Multiplying that by the inverse of the area of the *end piece of the cylinder in which the charge resides* yields a charge per unit volume of $-Q/(\pi(2a)^2 - \pi a^2) = Q/3\pi a^2 L$. With that, Gauss's Law yields:




NOTE: The hollow's radius should be "a" while the outside radius is "2a". The sketch is NOT to scale to afford you a better look at the Gaussian cylinder, etc.

$$\int \mathbf{E} \bullet d\mathbf{S} = \frac{q_{enclosed}}{\varepsilon_o}$$

$$\Rightarrow \quad E(2\pi r L) = \frac{\lambda L + \int_a^r \rho dV}{\varepsilon_o}$$

$$\Rightarrow \quad E = \frac{\lambda L + \int_{c=a}^{r} \left(\frac{-Q}{3\pi a^2}\right)(2\pi c L dc)}{(2\pi r L)\varepsilon_o}$$

$$\Rightarrow \quad E = \frac{\lambda + \left(\frac{-2Q}{3a^2}\right)\int_{c=a}^{r} (c)dc}{2\pi\varepsilon_o r}$$

$$\Rightarrow \quad E = \frac{\lambda + \left(\frac{-2Q}{3a^2}\right)\left[\frac{c^2}{2}\right]_a^r}{2\pi\varepsilon_o r}$$

$$\Rightarrow \quad E = \frac{\lambda + \left(\frac{-Q}{3a^2}\right)\left(r^2 - a^2\right)}{2\pi\varepsilon_o r}.$$

Evaluating this at $r = 5a/4$, we get:

$$E = \frac{\lambda + \left(\frac{-Q}{3a^2}\right)\left(r^2 - a^2\right)}{2\pi\varepsilon_o r}$$

$$\Rightarrow \quad 0 = \lambda + \left(\frac{-Q}{3a^2}\right)\left(\left(\frac{5a}{4}\right)^2 - a^2\right)$$

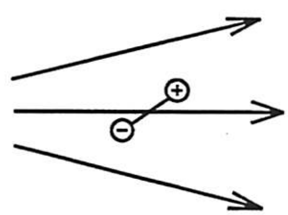
$$\Rightarrow \quad \lambda = \frac{3}{16}Q.$$

As this is approximately .188Q, this response is true.]

b.) Approximately .212Q coulombs per unit length. [Nope.]

c.) Approximately .590Q coulombs per unit length. [Nope.]

d.) None of the above. [Nope.]



**22.)** A dipole is placed in an electric field as shown. Over time, the dipole will:

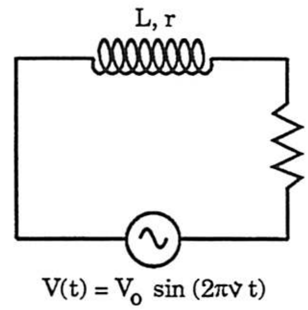
a.) Experience a constant acceleration of its center of mass toward the right and will experience a constant torque that motivates it to angularly accelerate in a clockwise direction. [The electric field in this situation is getting smaller as one moves to the right. That means that the force on the positive charge directed to the right (i.e., in the direction of the electric field lines) will be smaller than the force on the negative charge directed to the left. In other words, there will be a net force on the dipole that will accelerate its center of mass to the left. That, in itself, makes this statement false. There will also be a torque on the dipole that will vary in magnitude depending not only upon the angular position of the dipole relative to the electric field (when the dipole is aligned with the electric field, the torque will be zero--when the dipole is perpendicular to the electric field, the torque will be a maximum), but also relative to the actual forces applied (they will vary depending upon where the dipole's ends are positioned at a particular instant). That torque will be in the clockwise direction. Still, this response is false.]

b.) Experience no acceleration of its center of mass but will experience a varying torque that motivates it to angularly accelerate in a clockwise direction. [Acceleration will be experienced, so this response is false.]

**c.) Experience a varying acceleration of its center of mass toward the left and will experience a varying torque that motivates it to angularly accelerate in a clockwise direction.** [This is the one.]

d.) Experience no acceleration but will experience a varying torque that motivates it to angularly accelerate in a counterclockwise direction. [An acceleration will be experienced, so this response is false.]

e.) None of the above. [Nope.]




**23.)** Assume the inductor has no internal resistor-like resistance associated with it. If the frequency is doubled in this circuit:

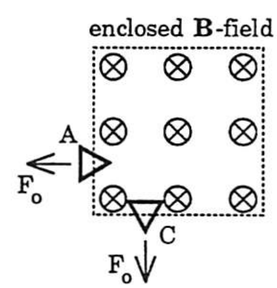
**a.) The resistive nature of the inductor will double, but the current will not halve.** [The resistive nature of an inductor is its inductive reactance. That quantity is mathematically equal to $2\pi\nu L$. Doubling the frequency will double the inductive reactance which, in turn, means the resistive nature of the inductor doubles. The first part of this response is true. As for the second part, this is a little bit trickier. The current in the circuit is governed by Ohm's Law. That is, $V_{RMS} = i_{RMS}Z$, where $Z$ is the circuit's impedance at the frequency of operation. We have already established that the inductive reactance doubles with a doubling of the frequency, but how does the circuit's net resistive nature--its impedance--change? The impedance expression for an RL circuit is $\sqrt{R_{net}^2 + (2\pi\nu L)^2}$. Because it is governed not only by the frequency dependent inductive reactance but also by the net resistance in the circuit, the impedance will increase when the frequency is doubled . . . but not by a factor of a two. As the impedance change is not double, the current will not halve and this response is true.]

b.) The resistive nature of the inductor will halve and the current will double. [Nope.]

c.) The resistive nature of the inductor will double and the current will double. [If nothing else, you wouldn't expect the resistive nature to go up and additionally have the current go up. The two are at the opposite ends of the pole--big current means small resistance, and vice versa. This response is false.]

d.) The resistive nature of the inductor will halve and the current will halve. [Nope.]

e.) None of the above. [Nope.]




**24.)** Two identical triangles are being pulled out of a bounded **B**-field. At a given instant, each is half in, half out. In both cases, an external force magnitude $F_o$ is being exerted on the triangles as shown.

a.) The accelerations of both will be the same, and the directions of the induced currents in both will be counterclockwise. [Nope.]

b.) The acceleration of $A$ will be greater than the acceleration of $C$, and identical induced currents will exist in both circuits in the clockwise direction. [The easiest part of this question is the determination of the direction of the induced current, so we'll do that first. In this case, the magnetic flux is decreasing, so the induced magnetic field will move to fight that decrease by orienting itself *in the same direction as* the external field. The induced coil current that will effect a magnetic field *through the coil* in that direction will be clockwise. This response is false.]

c.) The acceleration of *A* will be less than the acceleration of *C*, and identical induced currents will exist in both circuits in the clockwise direction. [This has the correct direction for the induced current, so we now need to look at the other part of the question. Along with the external force $F_o$, there will be an induced retarding force acting on each coil (that force will be generated when the induced EMF created by the changing magnetic flux motivates an induced current in the coil--that current will interact with the external **B**-field creating a force on the wire that *opposes* the removal of the wire from the magnetic field). Due to their geometry, triangle *A's* change of flux will be smaller than triangle *C's* (*C* will have the larger *area change per unit time*), so its induced current will be smaller. Also, *C* has more wire in the magnetic field. Put all together, the force on *C* will be greater than the force on *A*. We know that the triangle that experiences the greatest retarding force will accelerate the least. That triangle is *C*. This response is false.]

d.) The acceleration of *A* will be greater than the acceleration of *C*, and identical induced currents will exist in both circuits in the counterclockwise direction. [The direction of the induced current will be clockwise, and the currents will not be the same. This response is false.]

e.) The acceleration of *A* will be less than the acceleration of *C*, and identical induced currents will exist in both circuits in the counterclockwise direction. [At the very least, the acceleration of *A* will be greater than *C*. This response is false.]

**f.) None of the above.** [This is the one.]

**25.)** A parallel plate capacitor is charged completely, then isolated. The plate's area is then doubled, and the distance between its plates is halved. Once all of the changes are made:

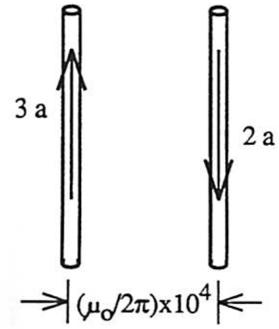
a.) The voltage across the plates will quadruple. [When a capacitor is charged, then isolated, what *cannot* change is the *charge on each plate* (there is nowhere for the charge to go). Keeping that in mind, we know that the capacitance for a parallel plate capacitor is $\varepsilon_o \frac{A}{d}$, and we know that $C = Q/V$. Putting it all together, if the plate area doubles and the distance between plates halves, the capacitance increases by a factor of four. The only way that this can happen, given that the charge cannot change, is if the voltage changes by a factor of one-quarter. This response is false.]

b.) The capacitance of the capacitor will quarter. [Nope.]

c.) The capacitance of the capacitor will halve. [Nope.]

**d.) None of the above.** [This is the one.]

**26.)** The 3 amp wire in the sketch feels a force due to the presence of the 2 amp wire. The direction of that force will be:




a.) Into the page. [At the 3 amp wire, the direction of the magnetic field produced by the 2 amp wire will be *into* the page (*right thumb rule*). Using ***iLxB***, the direction of force on the upward moving 3 amp current due to the 2 amp wire's magnetic field into the page will be *to the left*. This response is false.]

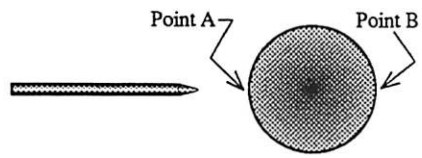
b.) Out of the page. [Nope.]

c.) To the right. [Nope.]

**d.) To the left.** [This is the one.]

e.) None of the above. [Nope.]

**27.)** Positive charge is uniformly distributed throughout a pointed rod made of insulating material. The rod is brought in close to an electrically neutral conducting ball as shown. Both *Point A* and *Point B* are *very close* to the ball.



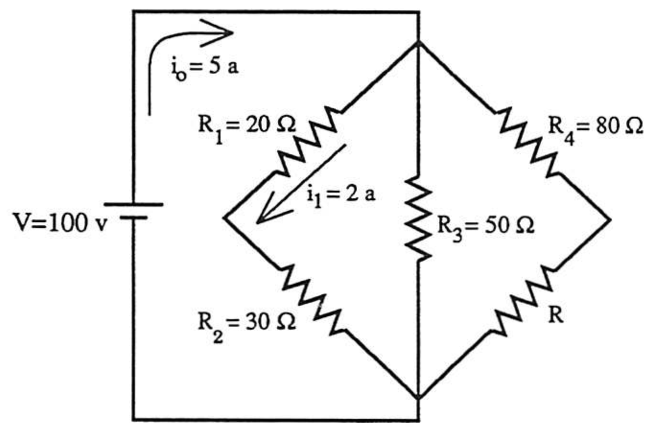

a.) The net electric field at *Point A* will be to the right, and the electric field at *Point B* will be to the left. [The presence of positive charge on the rod will motivate electrons in the conductor to migrate to that side of the sphere. In close on that side, therefore, the electric field will be generated by a negatively charged surface. As the electric field due to negative charge is in toward the surface, the direction of the field will be to the right. On *Point B*'s side of the sphere, there will be a preponderance of positive charge left due to the electron migration toward the other side of the ball. The electric field on that side will be that of a positively charged surface. As positive charges create electric fields that leave a surface, the electric field at *B* will be to the right. As such, this response is false.]

b.) The net electric field at *Point A* will be to the left, and the electric field at *Point B* will be to the left. [Nope.]

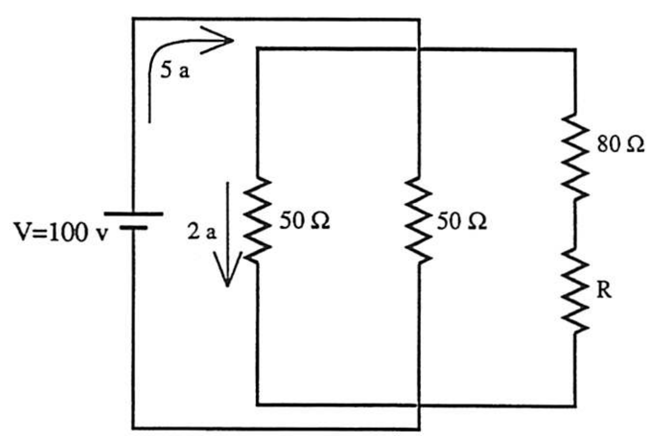
**c.) The net electric field at *Point A* will be to the right, and the electric field at *Point B* will be to the right.** [This is the one.]

d.) The net electric field at *Point A* will be to the left, and the electric field at *Point B* will be to the right. [Nope.]

**28.)** Given the information shown in the circuit, determine *R*.



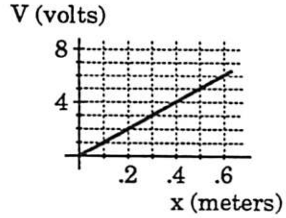

**a.) 20 Ω.** [This is deceptively easy if you see the trick and use your head. Combine the series resistors in the left branch of the parallel combination (i.e., the 20 Ω and 30 Ω branch) and re-draw the circuit. Due to the fact that the voltage in each branch of the parallel combination must be the same, and noticing that the center and left branch each have the same equivalent resistance, those branches must have the same current (2 amps each). We know that total current moving through the parallel combination is 5 amps. With a total of 4 amps passing through the two 50 Ω branches, we are left with 1 amp moving through *R's* branch. This is *half* of the current through either of the 50 Ω branches, so the resistance in *R's* branch must be *twice* that of either of the 50 Ω branch, or 100 Ω's total. The known resistor in *R's* branch is equal to 80 Ω's, so *R* must equal 20 Ω's. This response is true.]




b.) 40 Ω. [Nope.]

c.) 80 Ω. [Nope.]

d.) None of the above. [Nope.]




**29.)** An electrical potential field along the *x-axis* is defined by the graph shown. The electric field at $x = .2$ *meters* is:

a.) (10) **i** newtons/coulomb. [The relationship between an electric field and its associate electrical potential field is summarized in the relationship $\mathbf{E} = -\nabla V$. In a one-dimensional situation, the *rate of change* of the electrical potential function (its derivative) equals *minus* the electric field function (if this were more than one dimension, you would have to use the del operator and unit vectors). When dealing with a graph, the derivative of the graphed function ($V$ in this case) defines the *slope* of the function at any point. For our electrical potential graph, the slope is a constant equal to *+(2 volts)/(.2 meters) = 10 volts/meter*. A *volt/meter* is the same as a *newton/coulomb*, so this response looks like it should be true. Unfortunately, the electric field is not equal to the derivative of the electrical potential, it is equal to *minus* the derivative of the electrical potential. As such, the solution to the question is *-10 volts/meter*, and this response if false.]

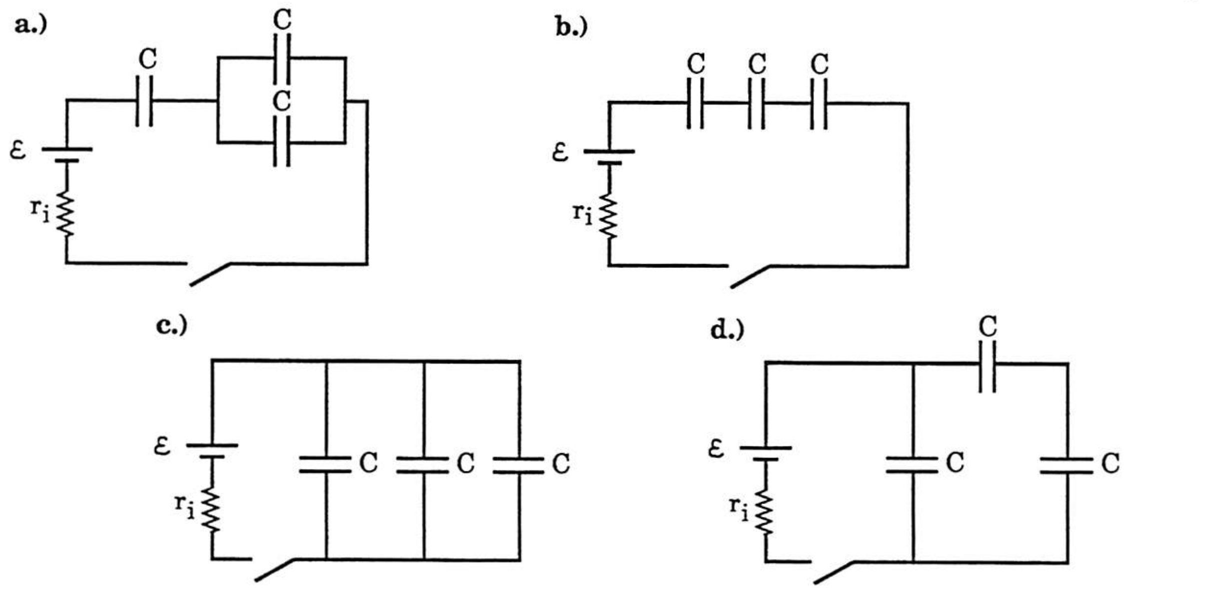
**b.) (-10) i newtons/coulomb.** [This is the one.]

c.) (-2) **i** newtons/coulomb. [If you used the graph to determine the electrical potential function (it was $10x$), then evaluated the resulting function between $x = 0$ where the fields are zero and $x = .2$ (the point of interest), you got this incorrect response.]

d.) (-.4) **i** newtons/coulomb. [If you took the integral of the electrical potential function, then evaluated it at $x = .2$ *meters*, you came up with this answer. It is incorrect.]

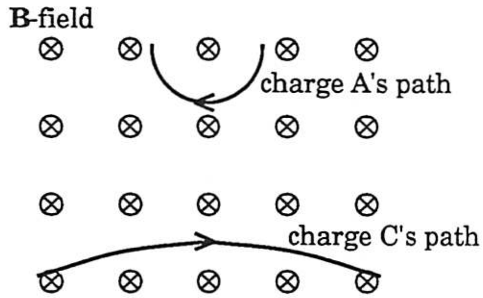
e.) There is not enough information to tell. [Nope.]

**30.)** In the circuit below, each of the capacitors characterized by $C$ is the same size and is initial-ly uncharged. The power supply's EMF is $\varepsilon$ while its very small internal resistance is $r_i$. At $t = 0$, all of the switches are closed. Which circuit will draw the most initial current from the battery?




a.) Circuit a. [This is a bit tricky. The uncharged capacitors act like shorts when the switch is thrown. That is, current effectively passes through them without any resistance to charge flow whatsoever. As such, we could as well remove all of the capacitors from the circuit and replace them with simple wires. As wires are theoretically assumed to have no effective resistance, this means that just as the switch is being thrown, the only resistance in the circuit is the internal resistance associated with $r_i$. As this is the same for each circuit, ALL circuits will have the same initial current. In other words, this circuit will do the job, but there are others.]

b.) Circuit b. [From above, this is not the only circuit to satisfy the situation.]
c.) Circuit c. [From above, this is not the only circuit to satisfy the situation.]
d.) Circuit d. [From above, this is not the only circuit to satisfy the situation.]
**e.) There is more than one circuit that will draw the maximum current.** [This is the one.]




**31.)** Two charges of equal mass move in the plane of the paper with the same kinetic energy. As they pass through a magnetic field, they are observed. A sketch of the situation is shown to the right. From observation, it can be deduced that:

a.) *Charge C* is negative, *charge A* is negative, and *charge C* is larger in magnitude than *charge A*. [We can determine whether, say, *charge A* is positive or negative by using $\boldsymbol{F} = q\boldsymbol{v}x\boldsymbol{B}$ and the right hand rule to determine the force direction on *charge A* <u>assuming *charge A* was positive</u> (remember, the equation $\boldsymbol{F} = q\boldsymbol{v}x\boldsymbol{B}$ is for a positive charge--a negative charge will feel a force in the opposite direction predicted by that cross product). Looking at the charge's path as it exists at the arrow on the sketch, the *right hand rule* suggests that a downward force would be felt by a positive charge in that situation. *Charge A* is clearly not accelerating in a downward arc, therefore it must be a negative charge. Following similar thinking on *charge C*, we find that that charge is also negative. So far, this response is OK. To determine which charge is larger, we need to derive a relationship between the *charge magnitude* and *path radius* of a charge moving in a fixed magnetic field when the velocity is held constant (we know that the two charges have the same velocity as both have the same mass *and* kinetic energy). To do this, we need to remember that magnetic forces are centripetal in nature. As such, the simplest *charge circling in a magnetic field* situation possible can be characterized as $qvB = m(v^2/r)$. With $q$, $B$, and $v$ all constant, we can see that the relationship between $q$ and $r$ is inverse in nature. In other words, the larger the charge is, the smaller the circle. As *charge A's* circle is smallest, *charge A's* charge must have the largest magnitude. As such, this response is false.]

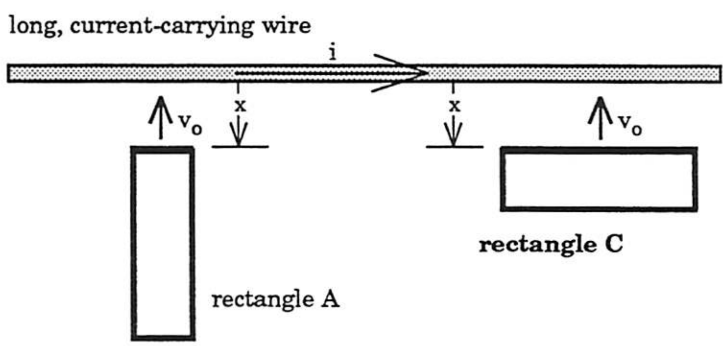
b.) *Charge C* is negative, *charge A* is positive, and *charge C* is larger in magnitude than *charge A*. [The charge types are off and this response is false.]

c.) *Charge C* is positive, *charge A* is negative, and *charge C* is smaller in magnitude than *charge A*. [The charge types are off and this response is false.]

d.) *Charge C* is positive, *charge A* is positive, and *charge C* is smaller in magnitude than *charge A*. [The charge types are off and this response is false.]

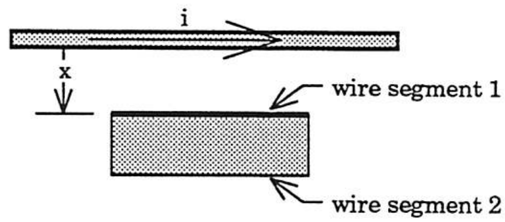
**e.) None of the above.** [This is the one.]

**32.)** Two identical rectangular loops are found a distance $x$ *units* from a long, current-carrying wire. Each rectangle is forced to approach the current-carrying wire with the same constant velocity (see sketch and ignore gravity).



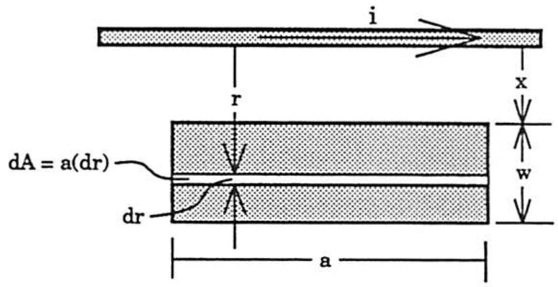
a.) The directions of the induced currents in both are clockwise, and the force required to move *rectangle A* at a given constant velocity is greater than the force required to move *rectangle C* at that same velocity. [The long wire is producing a magnetic field that is *into* the page in the region of the loops (use the *right thumb rule* to determine this). As the coils approach the long wire, the magnetic flux increases. That means the induced current sets up so as to create an induced magnetic field through the coils that is *out of the page*. To do that, the current must flow counterclockwise. From this, this is false.]

b.) The direction of the induced current in both is counterclockwise, and the force required to move *rectangle A* at a constant velocity is greater than the force required to move *rectangle C* at that same velocity. [This is the correct direction for the induced current. What about the second part of the response? My suspicion is that everyone's best guess would be that the force required to make *rectangle C* move with a given constant velocity would be greater than the force needed to make *rectangle A* move with that same velocity. After all, the shape of the *C* suggests that the magnetic flux through it will be greater than through *A*. That should mean a greater flux change and, as a consequence, a larger induced current. Unfortunately, there is a potential snag. There will be *two* forces on each coil the resultant of which will determine the ease or difficulty of moving the coil toward the long wire. The first force will be on the segment of wire designated *segment 1* in the auxiliary sketch. That force will be equal to ***iLxB***, where *i* is the induced current in the coil, *L* is the length of *segment 1*, and *B* is the magnitude of the external magnetic field a distance *x units* from the long wire (this **B**-field value will be $\frac{\mu_o i_{longwire}}{2\pi x}$). The second force will be on *segment 2*, will have the same algebraic form as the first force, will be smaller in magnitude because the **B**-field is farther from the long wire, and will be in the opposite direction (do the cross product to see this). The *difference* between the two forces will equal the net force one has to overcome to make the wire move with a constant velocity. The question is, "Are the two segments so close together that the force on *segment 1* is almost the same as the force on *segment 2*?" If that is the case, subtracting the two will leaves us with practically no net force to overcome. In *rectangle A*, *segment 2's* counterpart is so far away from the long wire that the force it will produce will be nil. On the other hand, the induced current in *coil A* will not be very large, the force producing segments are not very long, and the net effect may be very little net force required to make that coil move, either. Which requires the most force, or are they the same? As awful as this might seem, we really need to do the problem to see. And although I would most probably skip a question like this on an A.P. test, we *are* doing this for your own good. Here goes:




We will deal with *rectangle C* first. We need to use the expression $\boldsymbol{F} = \boldsymbol{iLxB}$ on both horizontal sections of wire to determine the amount of force each section feels due to the induced current's interaction with the external magnetic field. That means we need a general expression for the magnetic field in the region of the coils, we need the length of each wire section, and we need to determine the magnitude and direction of the induced current in the coil. Normally, we would start with Ampere's Law to determine the magnetic field expression. Fortunately, we don't have to do this as we know the field expression for a long, current-carrying wire. It is $\frac{\mu_o i_{longwire}}{2\pi r}$, where *r* is the distance between the wire and the point of interest. With that information, we can turn our attention to the induced current.

We can assume that the resistance in the rectangular coil is $R$. If we could determine the induced EMF in the coil, we could use $\varepsilon = i_{induced}R$ to determine the induced current in the coil. To get the induced EMF, we need to derive a general expression for the magnetic flux through the coil when the coil is some arbitrary distance $x$ units from the long wire. Unfortunately, we can't just do $BA \cos \theta$ to get the flux because the external **B**-field varies as we get farther and farther from the long wire, Instead, we need to define a differential area $dA$ through which the magnetic field is the same throughout, determine the differential flux $d\phi_m$ through that differential area, then integrate to determine the total magnetic flux $\phi_m$ through the coil's overall area.




The sketch defines the variables needed. Noting that as the magnetic field vector and area vector are in the same direction, the cosine term in the dot product becomes *1* and the magnetic flux calculation looks like:

$$\phi_m = \int \mathbf{B} \bullet d\mathbf{A}$$

$$\Rightarrow \quad \phi_m = \int_{r=x}^{x+w} \left( \frac{\mu_o i_{wire}}{2\pi r} \right) (a(dr))$$

$$\Rightarrow \quad \phi_m = \frac{\mu_o i_{wire} a}{2\pi} \int_{r=x}^{x+w} \left( \frac{1}{r} \right) dr$$

$$\Rightarrow \quad \phi_m = \frac{\mu_o i_{wire} a}{2\pi} \ln(r) \Big|_{r=x}^{x+w}$$

$$\Rightarrow \quad \phi_m = \frac{\mu_o i_{wire} a}{2\pi} [\ln(x+w) - \ln(x)]$$

With the magnetic flux expression, we can determine the induced EMF by taking $-N$ times the time derivative of the magnetic flux function. Noting that taking the time derivative of a function of $x$ requires the use of the *Chain Rule* (i.e., $df(x)/dt = [df(x)/dx]\,[dx/dt] \ldots$), we can write:

$$\varepsilon = -N \frac{d\phi_m}{dt}$$

$$\Rightarrow \quad \varepsilon = -N \frac{d\left( \frac{\mu_o i_{wire} a}{2\pi} [\ln(x+w) - \ln(x)] \right)}{dt}$$

$$\Rightarrow \quad \varepsilon = -\frac{\mu_o N i_{wire} a}{2\pi} \frac{d([\ln(x+w) - \ln(x)])}{dt}$$

$$\Rightarrow \quad \varepsilon = -\frac{\mu_o N i_{wire} a}{2\pi} \left[ \left( \frac{d[\ln(x+w) - \ln(x)]}{dx} \right) \left( \frac{dx}{dt} \right) \right]$$

$$\Rightarrow \quad \varepsilon = -\frac{\mu_o N i_{wire} a}{2\pi} \left[ \left( \frac{1}{(x+w)} - \frac{1}{x} \right) (v) \right]$$

With the induced EMF, the induced current becomes:

$$i_{induced} = \frac{\varepsilon}{R}$$

$$\Rightarrow \quad i_{induced} = \frac{-\frac{\mu_o N i_{wire} a}{2\pi}\left[\left(\frac{1}{(x+w)} - \frac{1}{x}\right)(v)\right]}{R}$$

The forces on the two horizontal sections will be in opposite directions (the force on the section nearest the long wire will be away from the wire in the negative direction). The magnitude of each will be determined using $i_{induced}LxB$. Adding those two forces vectorially yields:

$$F_{net} = -i_{induced}LxB_{nearsection} + i_{induced}LxB_{farsection}$$

$$\Rightarrow \quad F_{net} = -\left(\frac{\mu_o N i_{wire} a}{2\pi R}\left[\left(\frac{1}{x+w}\right)-\left(\frac{1}{x}\right)\right]v\right)[a]\left[\frac{\mu_o i_{wire}}{2\pi x}\right] + \left(\frac{\mu_o N i_{wire} a}{2\pi R}\left[\left(\frac{1}{x+w}\right)-\left(\frac{1}{x}\right)\right]v\right)[a]\left[\frac{\mu_o i_{wire}}{2\pi (x+w)}\right]$$

$$\Rightarrow \quad F_{net} = -\frac{\mu_o^2 N i_{wire}^2 a^2 v}{4\pi^2 R}\left[\left(\frac{1}{x+w}\right)-\left(\frac{1}{x}\right)\right]^2$$

The only difference between this force expression and the one derived for *rectangle A* is in the fact that for *rectangle A*, the *1/(x+w)* term becomes *1/(x+a)*. As $a > w$, and as both show up in the denominator, the net force on *C* will be greater than the net force on *A*, and it looks like our *best guess* back when we started the problem would have been a good bet. This response is false.]

c.) The direction of the induced current in both is clockwise, and the force required to move *rectangle A* at a constant velocity is less than the force required to move *rectangle C* at that same velocity. [Nope.]

**d.) The direction of the induced current in both is counterclockwise, and the force required to move *rectangle A* at a constant velocity is less than the force required to move *rectangle C* at that same velocity.** [This is the one.]

e.) There is no need for extra force to push the coils toward the wire as there is no gravity in the problem. [There will be a force due to the interaction between the induced current and the long wire's magnetic field. This response is false.]

**33.)** A .5 kg mass has a 10 coulomb charge on it. It is placed at *Point A* in a constant electric field and released from rest, freely accelerating to *Point B*. The electrical potential of *B* is 40 volts. The mass's velocity is calculated and determined to be equal to $v_1$. If the charge had accelerated twice the distance between *A* and *B*:

a.) The final velocity would have doubled to $2v_1$. [The way to approach this problem is through energy considerations. A doubling of the distance means a doubling of the amount of work done by the field. It also means a doubling of the electrical potential energy expended during the acceleration and a doubling of the kinetic energy change. As kinetic energy is a function of the square of the velocity, doubling the kinetic energy means increasing the velocity by $(2)^{1/2}$. This response is false. NOTE: It is true that because the electric field is constant, the charge's acceleration will be constant. A common error, though, is to assume that with a constant acceleration, a doubling of distance traveled will effect a doubling of the body's velocity. This is not true. If the *time* of travel had doubled, then it would have been true.]

b.) The final velocity would have quadrupled to $4v_1$. [Nope.]

c.) The final velocity would have halved to $v_1/2$. [Nope.]

**d.) None of the above.** [This is the one.]

**34.)** A 100 µf capacitor is charged by a constant 2 mA current. How long will it take for the voltage across the capacitor to reach 40 volts?

a.) .5 seconds. [Part of the test of this problem is figuring out the notation. You should know that *mA* stands for *milliamp*, or $10^{-3}$ *amps*. You should also know that the symbol *µf* stands for microfarads, or $10^{-6}$ *farads*. As for the question, about the only expression you know that has to do with current *and* includes *time* is the definition of current, or $i = q/t$ (note that $q$ in this expression is the amount of charge that passes by a point in the circuit during time $t$). With that definition, we can relate the amount of charge that gathers on the capacitor's plates in time $t$ with the relationship $q = it$. As the capacitor's capacitance will be $C = q/V$, we can additionally write $C = (it)/V$. Putting in our numbers, we get $t = CV/i = (100x10^{-6}\ farads)(40\ volts) / (2x10^{-3}\ amps) = 2x10^{-2}\ seconds$. This response is false.]

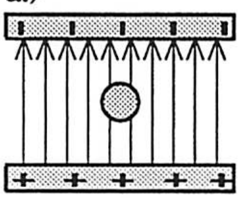
b.) .1 second. [Nope.]

**c.) .02 seconds.** [This is the one.]

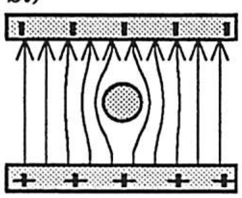
d.) None of the above. [Nope.]

**35.)** An uncharged sphere made of a covalently bonded substance is placed between oppositely charged plates (see sketch). The electric field lines between the plates will look like:

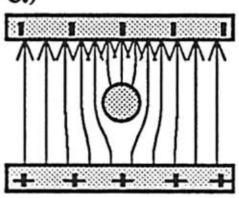
**a.)**



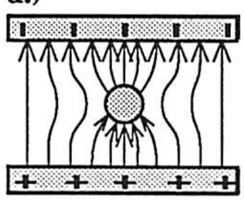
**b.)**



**c.)**



**d.)**



**e.)** None of these.

[**Commentary:** This question asks you to think about what happens to an insulator when it is placed in the relatively uniform electric field (i.e., the one produced by the plates). In that case, the Van der Waal's effect will make the top side of the ball (i.e., the side closest to the negative plate) electrically positive while making the bottom side of the ball electrically negative. This polarization will alter the electric field lines from a simply, parallel plate situation (without the sphere) depicted in *Response a* to the situation shown in ***Response d***. That is the one.]

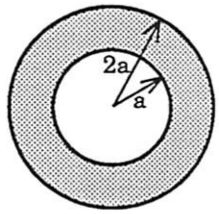
**36.)** A variable power supply produces an AC voltage equal to $5\ sin\ (4\pi t)\ volts$.

a.) The peak to peak voltage for this source is 5 volts, and the RMS voltage is 3.5 volts. [The phrase *peak to peak* is really a misnomer. It really should be *peak to trough*. Nevertheless, peak to peak values are always twice the amplitude. In this case, that would be 10 volts, and this response is false.]

**b.) The peak to peak voltage for this source is 10 volts, and the RMS voltage is 3.5 volts.** [This has the correct *peak to peak* voltage. What about the second part? The RMS voltage is the equivalent DC voltage required to provide to the circuit the same *power* the AC source is providing. As such, RMS values are based on power equivalence. They are numerically equal to .707 of the amplitude of the function you are working with (there are RMS values for currents, too). In this case, $V_{RMS} = .707V_o = .707(5\ volts) = 3.5\ volts$. This response is true.]

c.) The peak to peak voltage for this source is 10 volts, and the RMS voltage is 7.07 volts. [Nope.]

d.) None of the above. [Nope.]




**37.)** Three identical, hollow spheres each of inside radius $a$ and outside radius $2a$ each have different charge distributions that place a given, *common amount of charge* throughout their respective volumes. Denoting the spheres with the letters $A$, $B$, and $C$, the volume charge density on $A$ is $k_1r$, where $k_1$ is a constant; the volume charge density on $B$ is $k_2/r$, where $k_2$ is a constant; and the volume charge density on $C$ is $\frac{k_3e^{-k_4r}}{r^2}$, where $k_3$ and $k_4$ are constants. If $E_A$, $E_B$, and $E_C$ denote the electric fields of $A$, $B$, and $C$, respectively, as evaluated at $x = 2a$ (i.e., on the outer surface) then:

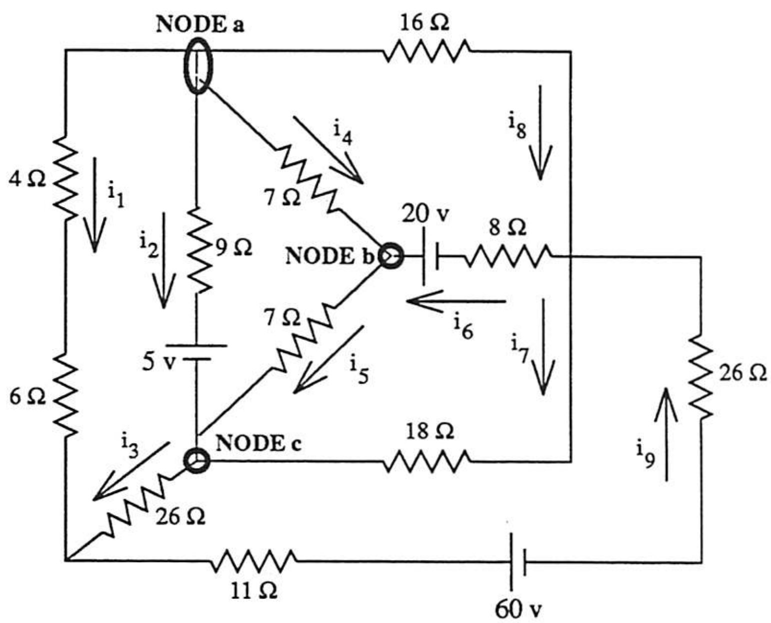
a.) $E_A > E_B > E_C$. [This is a typically tricky A.P. type question. It looks as though you will have to use the volume charge density functions to do the big-time Gauss's Law calculations required to derive full blown electric field functions for each sphere, then evaluate each of those functions at $x = 2a$. The clever souls in the crowd, on the other hand, will notice that the total amount of charge in each sphere is the same (it is just *how* the charge is distributed that is different--the *total* charge is the same in each case). That means that the Gaussian surface for each case will have the same *charge enclosed*, and each electric field will evaluate to the same magnitude at $r = 2a$. This response is false.]

b.) $E_A < E_B < E_C$. [Nope.]

c.) $E_A > E_C > E_B$. [Nope.]

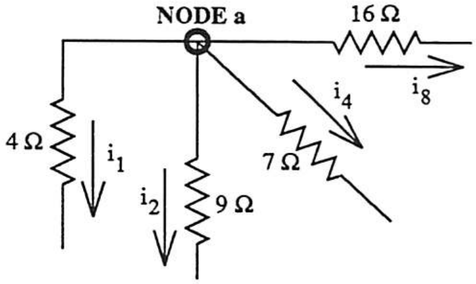
**d.)** $\mathbf{E_A = E_B = E_C}$. [This is the one.]

e.) None of the above. [Nope.]




**38.)** For the large circuit shown:

a.) $i_1 + i_2 + i_8 = 0$. [All of the nodes being alluded to in this and the following responses are highlighted in the auxiliary sketch (next page). This response is associated with *NODE a*. The response would have been fine if the current $i_4$ hadn't been ignored. NOTE: The 7 Ω branch (the branch whose current has been defined as $i_4$) is a part of *NODE a*. This is evident if you examine the alternate connection shown below to the

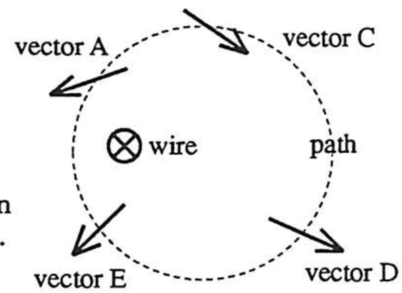



right. In any case, this response is false.]

b.) $i_5 + i_6 = i_4$. [*NODE b's* equation should be $i_6 + i_4 = i_5$. This response is false.]

**c.) $i_3 = i_2 + i_5 + i_7$.** [*NODE c's* equation is the equation shown. This response is true.]

d.) None of the above. [Nope.]




**39.)** A current-carrying wire is oriented so as to direct current *into* the page as shown in the sketch. An arbitrarily placed circle is positioned about the wire. Four vectors are defined on the circle. Which of the vectors accurately reflects the direction of the magnetic field as it exists where the vector crosses the circle?

a.) Vector A. [The *right thumb rule* suggests that the circulation of the magnetic field around the wire will be in the clockwise direction. That automatically eliminates any vector that is oriented in a counterclockwise direction . . . like the one in this response. This response is false.]

b.) Vectors A and E. [From above, this won't do.]

**c.) Vector C.** [Magnetic field lines generated by a current-carrying wire are always tangent to the circle centered on the wire that passes through the point of interest (the *right thumb rules* tells us whether that circulation is clockwise or counterclockwise). Drawing an imaginary circle through the point associated with *vector C* finds that vector *tangent to the imaginary circle*. As the general direction of the vector is in the direction of the magnetic field (as determined in *Response a*), this response is true. Are there others?]

d.) Vectors C and D. [Drawing an imaginary circle through the point associated with *vector D* does not find that vector tangent to the imaginary circle. As such, *vector D* is not in the direction of the wire's magnetic field at that point. This response is false.]

e.) Vector D. [A wire's **B**-field lines are *not* directed radially out from the wire.]

f.) None of the above. [Nope.]

**40.)** The magnitude of an electric field is defined by the expression $E = kr^2$, where $k$ is a constant. The electrical potential function that goes with this electric field function is:

**a.) $-kr^3/3$.** [If you kissed this possibility off immediately because the function is negative, that was a mistake. To see why, we have to do the math. The relationship between a *varying* electric field and its associated electrical potential field is

$V(r) - V(zero\ point) = -\int_{zero\ point}^{r} E \bullet dr = V(r)$. Noting that the electrical potential will be zero where the force is zero (at $r = 0$), this expression yields:

$V(r) - V(0) = -\int_0^r (kr^2\mathbf{r}) \bullet (dr\mathbf{r}) = -\int_0^r kr^2 dr = -\frac{kr^3}{3}$. The temptation is to look at this expression and think, *wait a minute, how can this be negative?* The point is that it doesn't matter whether it is negative. What matters is that if you take $\Delta V$ between any two points, *minus* the number you get will equal *the work per unit charge* done by the field in moving between the two points. That is, if you move a positive point charge from $r = 1$ *meter* to $r = 2$ *meters*, the work the field does should be positive (the force and displacement are in the same direction). According to our electrical potential expression, $W/q = -\Delta V = -[-k(2)^3/3 - k(1)^3/3] = +7k/3$, a positive number (just as expected). It is always good to be suspicious when you run into an expression that doesn't seem to make sense, but upon closer inspection, this one turns out OK. This response is true.]

b.) 2kr. [This is what you got if you took the derivative of the electric field function. If you thought that this was the way to go, you have things backwards. An electric field is related to the *spatial rate of change* of its associate electrical potential field (that is, $E$ is related to the *derivative of* $V$, not vice versa). This response is false.]

c.) $kr^2 \cos\theta$, where $\theta$ is the angle between the electric field vector and a line from the origin to the point in question. [This is blarney, a cheap attempt to try to fake you into thinking about the expression $\Delta V = -Ed\cos\theta$. I should be ashamed. This response is false.]

d.) None of the above. [Nope.]